Towards a Dialectical Approach to Abstract Argumentation Semantics

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Abstract

We consider abstract argumentation frameworks (AFs) and their semantics. The standard approach in the literature are the extension-based semantics, under which sets of jointly acceptable arguments are computed. While there exist alternatives, like labeling-based semantics, none of these approaches are able to model the procedural aspect of argumentation on the semantical layer. In this work, we present the sequence-based semantics, that compute sequences of minimally acceptable sets (called serialisation sequences), as a step towards a dialectical form of argumentation semantics.

1 Introduction

Formal Argumentation remains a important research area in knowledge representation and reasoning. In particular, on the topic of explainable artificial intelligence, argumentative methods have shown to be very promising approaches with numerous recent developments [Cyras et al., 2021]. The most prominent argumentation formalism continues to be the *abstract argumentation framework* (AF), where arguments are modeled as abstract entities and directed attacks represent conflicts between them [Dung, 1995].

Argumentation is inherently linked with dialectics [Rescher, 1977]. A central aspect of dialectical argumentation is the procedurality, i. e., the fact that arguments and counterarguments are brought forward one after another in alternating fashion. In abstract AFs this aspect is modelled well on the syntactical level via the directional attack relation between arguments. However, on the semantical layer the procedurality is typically lost in the existing approaches [Verheij, 1996].

In the literature, the most widespread approach are the extension-based semantics as introduced by [Dung, 1995]. In this method, the semantic conclusions of an AF are simply represented by sets of arguments (called extensions), without any consideration of the process of arriving at that conclusion. Alternatively, there is the labeling-based approach to semantics [Jakobovits and Vermeir, 1999, Caminada and Gabbay, 2009]. In this approach the semantic conclusions are total functions that distinguish between accepted, rejected and undecided arguments of the AF. However, in that case the underlying process of argumentation is also not represented. Notably, the ranking-based approach is also prominent in the literature [Bonzon et al., 2016]. In contrast to the former two approaches, this method computes a ranking of the arguments in the AF according to different criteria. While some ranking-based semantics do consider dialectical information of the AF [Amgoud and Ben-Naim, 2013], this is not apparent in the resulting ranking.

We consider the concept of *serialisability* of argumentation semantics [Thimm, 2022]. This method allows us to represent an extension as a sequence of minimally acceptable sets (called serialisation sequences), constituting the order in which the arguments must be accepted to form a semantically valid conclusion. All classical admissibility-based semantics of Dung can be characterised by this approach.

In this work, we aim to present the *sequence-based semantics*, formulated via the serialisation sequences, as a novel and powerful approach to semantics for abstract argumentation. Under this approach the underlying reasoning process of a semantic conclusion is represented directly in form of a sequence. We highlight, how this leads to a more expressive form of semantics and outline our plans on how to further analyse this method and generalise the approach to other prominent argumentation formalisms.

2 Method

We consider the *(abstract) argumentation framework* (AF) in the sense of [Dung, 1995].

Definition 1. *An* argumentation framework *(AF) is a pair $F = (A, R)$ where A is a finite set of arguments and $R \subseteq A \times A$ is the binary attack relation.*

For two arguments $a, b \in A$, we say that *a attacks b* iff aRb. For a set $S \subseteq A$ we define

$$S_F^+ = \{a \in A \mid \exists b \in S : bRa\}$$
$$S_F^- = \{a \in A \mid \exists b \in S : aRb\}$$

For two sets S and S' we write SRS' iff $S' \cap S_F^+ \neq \emptyset$.

2.1 Extension-based Semantics

We say that a set $S \subseteq A$ is *conflict-free* iff for all $a, b \in S$ it holds that $(a, b) \notin R$. A set S *defends* an argument $b \in A$ iff for all a with aRb there is $c \in S$ with cRa. Furthermore, a set S is called *admissible* (ad) iff it is conflict-free and S defends all $a \in S$.

The classical extension-based semantics for AFs are then defined by imposing further constraints on the admissible sets [Baroni et al., 2018]. In particular, an admissible set E is

- *complete* (co) iff for all $a \in A$, if E defends a then $a \in E$,
- *grounded* (gr) iff E is complete and $\subseteq$-minimal,
- *preferred* (pr) iff E is $\subseteq$-maximal,
- *stable* (st) iff $E \cup E_F^+ = A$,
- *strongly admissible* (sa) iff $E = \emptyset$ or each $a \in E$ is defended by a strongly admissible $E' \subseteq E \setminus \{a\}$.

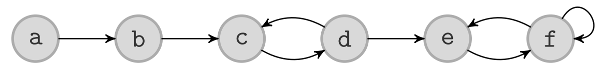

Figure 1: The AF F_1 from Example 1.

For the context of this work we consider ad to be a semantics itself. For any semantics σ let $\sigma(F)$ denote the set of σ-extensions of F.

Example 1. *Consider the AF F_1 in Figure 1. The AF F_1 has the three complete extensions $\{a\}$, $\{a, d\}$ and $\{a, c, e\}$. The set $\{a\}$ is also the unique grounded extension of F_1, while the latter two sets are the preferred extensions and only $\{a, c, e\}$ is stable. The strongly admissible sets of F_1 are $\emptyset$ and $\{a\}$.*

2.2 Sequence-based Semantics

Before we turn to the sequence-based approach to argumentation semantics, we first consider their building blocks, the *initial sets* [Xu and Cayrol, 2018].

Definition 2. *For $F = (A, R)$, a set $S \subseteq A$ with $S \neq \emptyset$ is called an* initial set *(is) if $S \in \mathsf{ad}(F)$ and there is no $S' \in \mathsf{ad}(F)$ with $S' \subsetneq S$ and $S' \neq \emptyset$.*

We denote with $\mathsf{is}(F)$ the set of initial sets of F. In general, we also distinguish between three types of initial sets [Thimm, 2022].

Definition 3. *For $F = (A, R)$ and $S \in \mathsf{is}(F)$, we say that*

1. *S is* unattacked *($\mathsf{is}^{\not\leftarrow}$) iff $S_F^- = \emptyset$,*
2. *S is* unchallenged *($\mathsf{is}^{\not\leftrightarrow}$) iff $S_F^- \neq \emptyset$ and $\nexists S' \in \mathsf{is}(F)$ with $S'RS$,*
3. *S is* challenged *($\mathsf{is}^{\leftrightarrow}$) iff $\exists S' \in \mathsf{is}(F)$ with $S'RS$.*

Furthermore, denote with $\mathsf{is}^{\not\leftarrow}(F)$, $\mathsf{is}^{\not\leftrightarrow}(F)$ and $\mathsf{is}^{\leftrightarrow}(F)$ the unattacked, unchallenged and challenged initial sets of F respectively. Secondly, we also require the notion of the *reduct* [Baumann et al., 2020] to define the sequence-based semantics.

Definition 4. *For $F = (A, R)$ and $S \subseteq A$, the S-* reduct *is defined as $F^S = (A', R')$ with*

$$A' = A \setminus (S \cup S_F^+)$$
$$R' = R \cap (A' \times A')$$

Example 2. *We continue Example 1 and consider again the AF F_1 in Figure 1. The initial sets of F_1 are $\{a\}$ and $\{d\}$. The set $\{a\}$ is an unattacked initial set while $\{d\}$ is unchallenged initial. The $\{a\}$-reduct $F_1^{\{a\}}$ contains the arguments $\{c, d, e, f\}$ and the respective attacks. Notably, in $F_1^{\{a\}}$ there are the initial sets $\{c\}$ and $\{d\}$, both of which are challenged initial.*

Finally, as the corresponding notion to the admissible sets, we define the *serialisation sequence* $\mathcal{S}$ as a series of initial sets of the respective reducts [Blümel and Thimm, 2022].

Definition 5. *A* serialisation sequence *for $F = (A, R)$ is a sequence $\mathcal{S} = (S_1, \ldots S_n)$ with $S_1 \in \mathsf{is}(F)$ and for each $2 \leq i \leq n$ we have that $S_i \in \mathsf{is}(F^{S_1 \cup \cdots \cup S_{i-1}})$.*

For some serialisation sequence $\mathcal{S} = (S_1, \ldots S_n)$ we also define the corresponding admissible set as $\hat{\mathcal{S}} = S_1 \cup \cdots \cup S_n$. Essentially, a serialisation sequence can be understood as a construction of its corresponding set by solving a series of atomic conflicts.

Now, analogously to the extension-based approach, we define the sequence-based semantics for AFs.

Definition 6. *Let $F = (A, R)$ be an AF. We say that a serialisation sequence $\mathcal{S} = (S_1, \ldots S_n)$ is*

- $\mathcal{S}$ complete *iff $\mathsf{is}^{\not\leftarrow}(F^{\hat{\mathcal{S}}}) = \emptyset$,*
- $\mathcal{S}$ grounded *iff for all S_i, $i = 1, \ldots, n$, it holds that $S_i \in \mathsf{is}^{\not\leftarrow}(F^{S_1 \cup \cdots \cup S_{i-1}})$ and $\mathsf{is}^{\not\leftarrow}(F^{\hat{\mathcal{S}}}) = \emptyset$,*
- $\mathcal{S}$ preferred *iff $\mathsf{is}(F^{\hat{\mathcal{S}}}) = \emptyset$,*
- $\mathcal{S}$ stable *iff $F^{\hat{\mathcal{S}}} = (\emptyset, \emptyset)$,*
- $\mathcal{S}$ strongly admissible *iff for all S_i, $i = 1, \ldots, n$, it holds that $S_i \in \mathsf{is}^{\not\leftarrow}(F^{S_1 \cup \cdots \cup S_{i-1}})$.*

For any semantics σ let $\mathfrak{S}_\sigma(F)$ denote the set of σ-serialisation sequences of F. For any AF F, we can show that every σ-extension has at least one corresponding σ-serialisation sequence and every σ-serialisation sequence of F corresponds to exactly one σ-extension [Thimm, 2022].

Theorem 1. *Let $F = (A, R)$ be an AF and $\sigma \in \{\mathsf{ad}, \mathsf{co}, \mathsf{gr}, \mathsf{pr}, \mathsf{st}, \mathsf{sa}\}$ is a semantics. If $S \in \sigma(F)$, then there exists some $\mathcal{S} \in \mathfrak{S}_\sigma(F)$ such that $S = \hat{\mathcal{S}}$. If $\mathcal{S} \in \mathfrak{S}_\sigma(F)$, then $\hat{\mathcal{S}} \in \sigma(F)$.*

Example 3. *We continue Example 2 and consider again the AF F_1 in Figure 1. We have the complete serialisation sequence $(\{a\}, \{c\}, \{e\})$ which is the only sequence corresponding to the complete extension $\{a, c, e\}$. On the other hand, for the complete extension $\{a, d\}$, we have two corresponding sequences for F_1: $(\{a\}, \{d\})$ and $(\{d\}, \{a\})$.*

3 Discussion

First, we introduce a way to represent the σ-serialisation sequences of an AF in a graphical form. For that, we represent a serialisation sequence $(S_1, \ldots S_n)$ as a sequence of transitions $\emptyset \xrightarrow{S_1} S_1 \xrightarrow{S_2} S_1 \cup S_2 \xrightarrow{S_3} \ldots \xrightarrow{S_n} S_1 \cup \cdots \cup S_n$. Every state $S_1 \cup \cdots \cup S_i$ for some $i = 1, \ldots, n$ represents a (partial) σ-extension of the AF that may be shared by multiple serialisation sequences. We assemble these transition sequences into a graph where the intermediate states are the nodes and the transitions are the directed edges. σ-extensions are highlighted in bold.

$$\begin{tikzcd}
 & \{a\} \arrow[r, "\{c\}"] \arrow[ddr, "\{d\}"] & \boldsymbol{\{a, c\}} \\
\emptyset \arrow[ur, "\{a\}"] \arrow[dr, "\{d\}"] & & \\
 & \{d\} \arrow[r, "\{a\}"] & \boldsymbol{\{a, d\}}
\end{tikzcd}$$

Figure 2: pr-serialisation of F_1 described in Ex. 3.

The resulting structure is a complete lattice over admissible sets of some AF F, cf. Figure 2. The least element is the empty set and maximal elements are always the preferred extensions of F. The edge labels of the lattice chains ending at a bold node then represent the σ-serialisation sequences of F.

In recent work, we showed that the sequence-based semantics are more expressive than the extension-based approach.For that, consider first the notion of equivalence of AFs based on their σ-extensions.

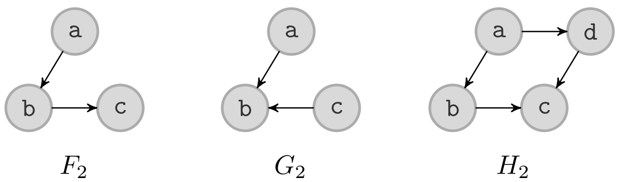

Figure 3: The argumentation frameworks F_2, G_2 and H_2, which have the same preferred extension: $\{a, c\}$.

Definition 7. *Let F and G be AFs. We say that F and G are* σ-extension equivalent*, written $F \equiv_\sigma G$, iff $\sigma(F) = \sigma(G)$.*

In [Bengel et al., 2024], we then introduced the notion of *σ-serialisation equivalence* under which two AFs are only considered equivalent iff they possess the same σ-serialisation sequences.

Definition 8. *Let F and G be AFs. We say that F and G are* σ-serialisation equivalent σ*, written as $F \equiv^{se}_\sigma G$, iff $\mathfrak{S}_\sigma(F) = \mathfrak{S}_\sigma(G)$.*

We have then shown that for any semantics σ, σ-serialisation equivalence does generally imply σ-extension equivalence [Bengel et al., 2024].

Theorem 2. *Let $\sigma \in \Sigma$ be a semantics. For any two AFs F and G, if $F \equiv^{se}_\sigma G$, then $F \equiv_\sigma G$.*

More importantly however, the other direction does not hold in general for most semantics, as shown by Example 4. Only for ad and sa do the two equivalence notions coincide. That means, the sequence-based semantics are generally more expressive than the corresponding extension-based semantics (and thus also the labeling-based version).

Example 4. *Consider the AFs F_2, G_2 and H_2 in Figure 3 and their* pr*-serialisation in Figure 4. We have that $F_2 \equiv^{se}_{pr} H_2$. However, it holds that $F_2 \equiv_{pr} G_2$ but $F_2 \not\equiv^{se}_{pr} G_2$. The same holds for G_2 and H_2.*

The principle-based approach to analyse the behaviour of extension-based semantics is a prominent part of the literature [van der Torre and Vesic, 2018]. We intend to establish a similar approach for sequence-based semantics. First steps for that have already been made in [Bengel and Thimm, 2022], where we introduced the *closure* property of serialisation sequences which is satisfied by a semantics σ iff every admissible serialisation sequence $\mathcal{S}$ is either a σ-sequence or there exists an extension of that sequence which is a σ-sequence. We have shown that the closure of σ implies that σ satisfies *directionality*.

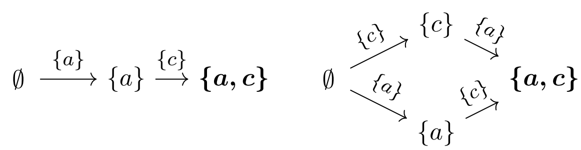

Figure 4: pr-serialisation of F_2, H_2 (left) and G_2 (right).

Furthermore, we are currently generalising the sequence-based approach to other argumentation formalisms, as already outlined in [Bengel, 2023]. In particular, that includes abstract dialectical frameworks (ADFs) [Brewka et al., 2013] where we define a serialisation sequence as a series of three-valued models. These sequences even describe, step by step, the process of both accepting and rejecting arguments.

Another important step in future work is considering structured argumentation. For that, we are particularly interested in assumption based argumentation (ABA) [Dung et al., 2009].

4 Conclusion

In this work, we considered the notion of *serialisability* of semantics, which provides a characterisation of the classical admissibility-based semantics in the form of serialisation sequences. We formulated these as a step towards a dialectical form of argumentation semantics, which we call *sequence-based semantics*. We introduced a graphical lattice representation of sequence-based semantics and showed that they are indeed more expressive than the existing approaches. Furthermore, we outlined future work to establish a principle-based analysis of sequence-based semantics and ongoing work to generalise the approach to other argumentation formalisms, namely ADFs and structured argumentation in the form of ABA.

Acknowledgements

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